Bibliography and Selected Quotes

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List of Topics Below

* Applications
* Mathematics
* History and Philosophy / Value of Quaternions
* Cognition
* Music

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Introduction Excerpt: “Albert Einstein found a use for four dimensions -- in order to make the speed of light constant for all inertial observers, space and time had to be united.”

Discussion and Conclusions Excerpt: “Quaternions have been applied in a new way as a means of representing pixel values in a colour image [25, 26, 28]. When colour images are processed, instead of individually manipulating the three real numbers representing the RGB values of the pixel, the triple is considered as a whole entity and processed as a whole quaternion number.”

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books.google.com/books?isbn=0486317730

* Quaternions came from Hamilton after his really good work had been done, and though beautifully ingenious, **have been an unmixed evil** to those who touched them in any way, including Clerk Maxwell.  
    
  (*Lord Kelvin, 1892, Letter to Heyward).*

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Anderson, R. and Joshi, G. (1993). Quaternions and the heuristic role of mathematics in physics. *Physics Essays*, 6, 308-319  
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* One of the most important ways development takes place in mathematics is via a process of generalization. On the basis of a recent characterization of this process we propose a principle that generalizations of mathematical structures that are already part of successful theories serve as good guides for the development of new physical theories.
* The principle is a more formal presentation and extension of a position stated earlier in this century by Dirac.
* **Quaternions form an excellent example of such a generalization and we consider a number of ways in which their use in physical theories illustrates this principle.**

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This would also be more compatible with the research of Dr. Wolpert who states that **everything about the brain is dependent on locomotion. Therefore I believe the emotional affects are not predicated on survival mechanisms or neurological circuitry hardwired in the Central Nervous System, but rather how the body and brain interplay on a much deeper level, specifically how to move the body from point A to point B.** Emotional experiences arise in the mind when the  brain generates neurological energies by tracking a subliminal beam of attention on the body’s physical center of gravity, that are likewise coupled to the movement of  external objects of attraction.

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<http://www.theorionproject.org/en/Bearden_ARPA-E.html>

Meanwhile, circa 1890 Tesla was hell-bent on giving the world "free EM energy from the active medium". Morgan, who had prepared the plan that would forever destroy Tesla, demanded to know from his scientific advisors if those Heaviside equations being considered for the new technology "electrical engineering" to be formed and taught in universities, still contained any of those confounded Tesla "energy from the active medium systems". [We point out that T. W. Barrett, a great higher group symmetry electrodynamicists and one of the cofounders of ultrawideband radar, in fact did a proper higher group symmetry electrodynamics (quaternion electrodynamics) analysis of some of Tesla's patented circuits, and proved that Tesla indeed could have done what he said. The reference is T. W. Barrett, "Tesla's Nonlinear Oscillator-Shuttle-Circuit (OSC) Theory," Annales de la Fondation Louis de Broglie, 16(1), 1991, p. 23-41. Barrett shows that EM expressed in quaternions allows shuttling and storage of potentials in circuits, and also allows additional EM functioning of a circuit that a conventional EM analysis simply cannot and will not reveal. He shows that Tesla’s patented circuits did exactly this].

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Bourbaki, Nicolas (1984) *from Wikipedia: When Was Matrix Multiplication invented?*<http://www.math.harvard.edu/~knill/history/matrix/>   
To give the history of linear algebra is a task that is as important as it is difficult.

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Boynton, R. M. (1979), *Human Color Vision*. Holt, Rinehart, and Winston, p. 232-238 (Chromatic Mechanisms in the Laterate Geniculate Nucleus)

*On the LGN, I found the reference I had once seen years ago, theoretical and empirical, for the LGN general chromatic color polarities-forming ability. These form the* ***basis, or axes, for color relations****.*

Excerpt from p. 234:

“The pioneering work in this field was done by R.L. DeValois and his collaborators. Almost all of the early work from his laboratory was undertaken with large diffuse fields of monochromatic light, to which LGN single units were found to be marvelously responsive. It proved possible to fit most of the results into a [polarities/oppositions] scheme essentially like that presented earlier in this chapter. Some cells were found that apparently carried a red-green code. By comparison with their level of activity in the dark, such a cell might, for example, discharge more vigorously in response to long wavelengths delivered to the eye and less so for short ones. DeValois called these “+R-G” cells…. Figure 7.12 is from a major study based on diffuse field stimulation and recordings from the LGN of the macaque monkey.…”

My interpretation, and I think also of Boynton, is that the LGN [a component of the thalamus] is, in effect, **injecting a structure –** an algebraic structure -- into the phenomenology that was not present at all in the monotonically increasing raw-data frequency input (equivalent to merely a kind of gray scale, just lower-higher relations, not polarities).

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Buneo, C.A. and Anderson, R.A. (2006). The posterior parietal cortex: sensorimotor interface for the planning and online control of visually guided movements. *Neuropsychologia, 44 (13)*, 2594-606  
<http://www.ncbi.nlm.nih.gov/pubmed/16300804>

### Abstract

We present a view of the posterior parietal cortex (PPC) as a sensorimotor interface for visually guided movements. Special attention is given to the role of the PPC in arm movement planning, where representations of target position and current hand position in an eye-centered frame of reference appear to be mapped directly to a representation of motor error in a hand-centered frame of reference. This mapping is direct in the sense that it does not require target position to be transformed into intermediate reference frames in order to derive a motor error signal in hand-centered coordinates. Despite being direct, this transformation appears to manifest in the PPC as a gradual change in the functional properties of cells along the ventro-dorsal axis of the superior parietal lobule (SPL), i.e. from deep in the sulcus to the cortical surface. Possible roles for the PPC in context dependent coordinate transformations, formation of intrinsic movement representations, and in online control of visually guided arm movements are also discussed. Overall these studies point to the emerging view that, for arm movements, the PPC plays a role not only in the inverse transformations required to convert sensory information into motor commands but also in 'forward' transformations as well, i.e. in integrating sensory input with previous and ongoing motor commands to maintain a continuous estimate of arm state that can be used to update present and future movement plans. Critically, this state estimate appears to be encoded in an eye-centered frame of reference.

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Capozziello, S. and Lattanzi, A. (2006). Geometrical and algebraic approach to central molecular chirality: A chirality index and an Aufbau description of tetrahedral molecules. *Chirality*, *18, 7,* 462-468*.* Special Issue: Proceedings from the Seventeenth International Symposium on Chirality (ISCD-17), Parma, Italy, 2005 <http://onlinelibrary.wiley.com/doi/10.1002/chir.20272/abstract>

Abstract:

On the basis of empirical Fischer projections, we develop an algebraic approach to the central molecular chirality of tetrahedral molecules. The elements of such an algebra are obtained from the 24 projections which a single chiral tetrahedron can generate in S and R absolute configurations. They constitute a matrix representation of the O(4) orthogonal group. According to this representation, given a molecule with n chiral centres, it is possible to define an “index of chirality χ ≡ {n, p}”, where n is the number of stereogenic centres of the molecule and p the number of permutations observed under rotations and superimpositions of the tetrahedral molecule to its mirror image. The chirality index not only assigns the global chirality of a given tetrahedral chain, but indicates also a way to predict the same property for new compounds, which can be built up consistently. Chirality 18:462–468, 2006. © 2006 Wiley-Liss, Inc.

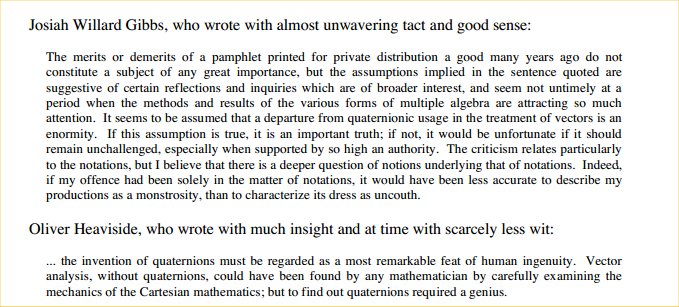
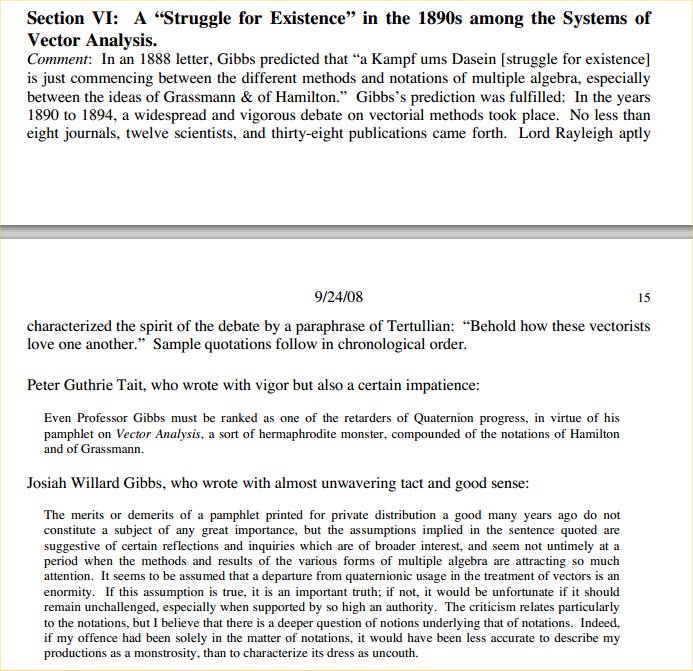
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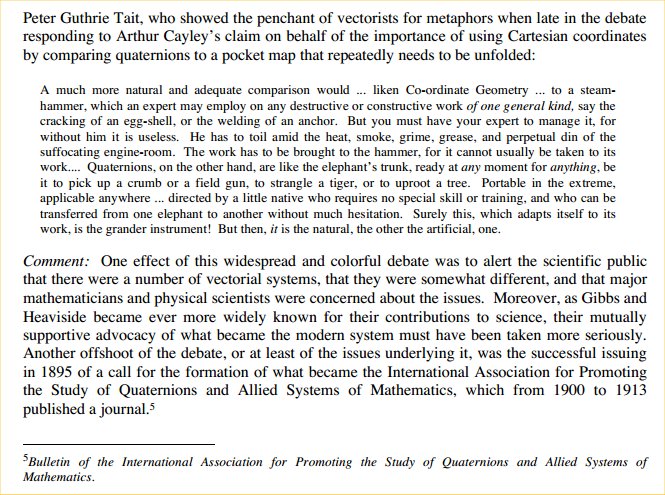
Chew, E. (2014). Mathemusical Conversations (*Singapore symposium announcement and call for papers*). Elaine Chew is Program Co-Chair.  
<http://elainechew-research.blogspot.com/2014/05/mathemusical-conversations-call-for.html>

[**Mathemusical Conversations**](https://sites.google.com/site/mathemusicalconversations) brings together world experts and emerging scholars in mathematics and music for discussions of how these fields intersect, and computational approaches combining the two. The joint international workshop between the Institute for Mathematical Sciences and the Yong Siew Toh Conservatory of Music of the National University of Singapore will take place February 13-15, 2015, at the conservatory. The workshop is organized in partnership with the Centre for Digital Music at Queen Mary University of London (UK), and the Sciences and Technologies for Music and Sound Research Lab at Ircam, CNRS, and Université Pierre et Marie Curie (France). This workshop will focus on research in performance and composition, which serves as the foundation both for understanding human creativity and for enabling future music technologies.  
  
Ancient philosophers such as Pythagoras, Leibniz, and Diderot have long drawn profound connections between mathematics and music. In the liberal arts education of medieval universities, music was considered one of the quadrivium—four mathematical science subjects taught after the foundational trivium—alongside arithmetic, geometry, and astronomy. Today, the connections between mathematics and music have renewed relevance. The rise of computing devices and the ensuing digitization of music has led to new ways to represent, create and perform music, to study and understand it, and to make scientific discoveries as to how music is made and why. At the core of this digital music revolution, are mathematical and computational techniques that drive scientific advances.   
  
**Almost every subfield of mathematics—including number theory, algebraic geometry, topology, geometric analysis, probability, statistics and mathematical physics—as well as modeling technique—including optimization, stochastic systems, game theory, and network analysis—has contributed to these advances in digital music research. Every step of the music process, from composition (including improvisation, a form of real-time composition) to performance, from conceptualization to perception and cognition, can be modeled and studied using mathematical and computational means.**  
The workshop's six thematic sessions featuring 24 internationally-renowned speakers, and concerts are designed for appeal to a broad audience. The sessions will focus on mathemusical engagement, geometries, canons, shaping performance, creativity, and educating the mathemusical. The event will interest researchers in the mathematical music sciences, as well as mathematicians and scientists in general, musicians interested in the mathematical sciences, and the general public.  
  
The Mathemusical Conversations workshop celebrates the understanding of the essence of music through mathematics, and the presentations and discussions will be published in a book to serve as a reference for the state of the art in mathematics and music thinking.

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Crowe, Michael J. (2002). The History of Vector Analysis  
(*talk based on the 1967/1994 book, at University of Louisville; gives quotes from the great debate*)  
<https://www.math.ucdavis.edu/~temple/MAT21D/SUPPLEMENTARY-ARTICLES/Crowe_History-of-Vectors.pdf>

The Great Debate (1890-1894):  




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Crowe, M. J. (1994). *A history of vector analysis: The evolution of the idea of a vectorial system*. Dover   
[store.doverpublications.com/0486679101.html](http://store.doverpublications.com/0486679101.html)

*(See Michael J. Crowe – Dover website)*

On October 16, 1843, Sir William Rowan Hamilton discovered quaternions and, on the very same day, presented his breakthrough to the Royal Irish Academy. Meanwhile, in a less dramatic style, a German high school teacher, Hermann Grassmann, was developing another vectorial system involving hypercomplex numbers comparable to quaternions. The creations of these two mathematicians led to other vectorial systems, most notably the system of vector analysis formulated by Josiah Willard Gibbs and Oliver Heaviside and now almost universally employed in mathematics, physics and engineering. Yet the Gibbs-Heaviside system won acceptance only after decades of debate and controversy in the latter half of the nineteenth century concerning which of the competing systems offered the greatest advantages for mathematical pedagogy and practice.  
This volume, the first large-scale study of the development of vectorial systems, traces he rise of the vector concept from the discovery of complex numbers through the systems of hypercomplex numbers created by Hamilton and Grassmann to the final acceptance around 1910 of the modern system of vector analysis. Professor Michael J. Crowe (University of Notre Dame) discusses each major vectorial system as well as the motivations that led to their creation, development, and acceptance or rejection.  
The vectorial approach revolutionized mathematical methods and teaching in algebra, geometry, and physical science. As Professor Crowe explains, in these areas traditional Cartesian methods were replaced by vectorial approaches. He also presents the history of ideas of vector addition, subtraction, multiplication, division (in those systems where it occurs) and differentiation. His book also contains refreshing portraits of the personalities involved in the competition among the various systems.  
Teachers, students, and practitioners of mathematics, physics, and engineering as well as anyone interested in the history of scientific ideas will find this volume to be well written, solidly argued, and excellently documented. Reviewers have described it as "a fascinating volume," "an engaging and penetrating historical study" and "an outstanding book (that) will doubtless long remain the standard work on the subject." In 1992 it won an award for excellence from the Jean Scott Foundation of France.

Reprint of the University of Notre Dame Press, 1967 edition.

p. 18 quote:

E. T. Bell’s view of Hamilton as a tragic figure certainly stemmed from the fact that he felt that quaternions are of little interest to modern mathematics. Bell was convinced that Hamilton was the victim of a monomaniacal delusion; he stated “that Hamilton’s deepest tragedy was neither alcohol nor marriage but his obstinate belief that quaternions held the key to the mathematics of the physical universe.” The problem of quaternions also stands behind much of what E. T. Whittaker wrote concerning Hamilton; Whittaker however passed over the problem by stressing Hamilton’s contributions to mathematical physics and by arguing that quaternions “may even yet prove to be the most natural expression of the new physics.”

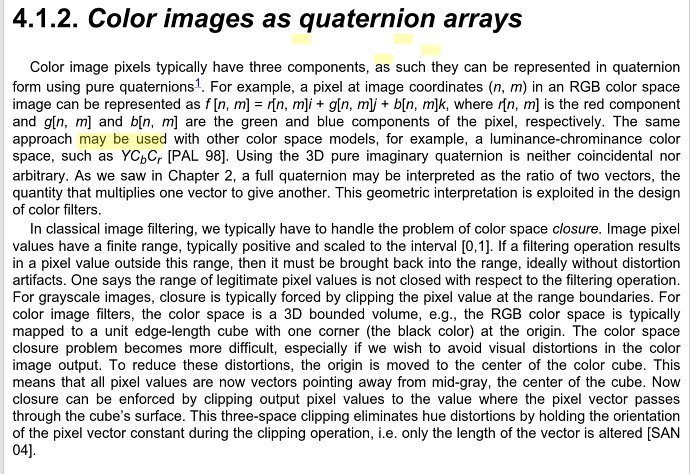
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De Leo, S. (1996). Quaternions and Relativity Theo ry. © 1996 American Institute of Physics, S0022-2488(96)01106-1  
<http://www.ime.unicamp.br/~deleo/Pub/p07.pdf>

We reformulate Special Relativity by a quaternionic algebra on reals. Using *real linear quaternions,* we show that previous difficulties, concerning appropriate transformations on the 3+1 space-time, may be overcome. This implies that a complexified quaternionic version of Special Relativity is a choice, and not a necessity.

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Ell, T., Le Bihan, N., and S. Sangwine (2014). *Quaternion Fourier Transforms for Signal and Image Processing*. Wiley  
[http://books.google.com/books?id=eGy8AwAAQBAJ&pg=PT63&dq=quaternion+color&hl=en&sa=X&ei=xj0gVM3wD5GqyASU4ILgCA&ved=0CB8Q6AEwAA#v=onepage&q=quaternion%20color&f=false](http://books.google.com/books?id=eGy8AwAAQBAJ&pg=PT63&dq=quaternion+color&hl=en&sa=X&ei=xj0gVM3wD5GqyASU4ILgCA&ved=0CB8Q6AEwAA)



Jaschke, A. C. (2012) Neuro-imaging reveals: Music changes brain. Tech The Future (website)  
<http://www.techthefuture.com/technology/neuro-imaging-reveals-music-changes-brains/>

But why music and why a central brain structure such as the thalamus?

Music is a multisensory stimulus, meaning it stimulates more than one sensory receptor in the brain. Its connection to the thalamus in interconnection with the claustrum, (a thin membrane like structure connecting different axonal pathways of one part of the cortex with another) creates a complex network of multisensory connections between the later two and the cortex (the cortex houses the four main lobes; frontal, parietal, occipital and temporal and can be seen as the surface of the brain).

[](http://www.techthefuture.com/wp-content/uploads/2012/07/Neuro-picture.png)

Tractograpy of the brainstem, thalamus and corpus callosum

This network implies a direct effect of music on the thalamus, which in return effects the whole of the brain and therefore can reach as far as influencing our everyday life, through stimulation of for example executive functions (e.g. planning, multi-tasking, learning, working memory, decision making and problem solving, to mention a few).

The above mentioned gateway, the thalamus, is responsible for directing incoming information -as in this case: music- to the necessary brain structures in the cortex. Furthermore it regulates the hormone household in the body through the Hypothalamus. Understanding where and how the thalamus directs music and what hormonal effect is attached to it, can have immense implications for disease management and treatment.

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Gibbs, J. W., Wikipedia excerpt (from Later Years)  
<http://en.wikipedia.org/wiki/Josiah_Willard_Gibbs>

Vectors:

From 1880 to 1884, Gibbs worked on developing the [exterior algebra](http://en.wikipedia.org/wiki/Exterior_algebra) of [Hermann Grassmann](http://en.wikipedia.org/wiki/Hermann_Grassmann) into a[vector calculus](http://en.wikipedia.org/wiki/Vector_calculus) well-suited to the needs of physicists. With this object in mind, Gibbs distinguished between the [dot](http://en.wikipedia.org/wiki/Dot_product) and [cross products](http://en.wikipedia.org/wiki/Cross_product) of two vectors and introduced the concept of [dyadics](http://en.wikipedia.org/wiki/Dyadics" \o "Dyadics). Similar work was carried out independently, and at around the same time, by the British mathematical physicist and engineer [Oliver Heaviside](http://en.wikipedia.org/wiki/Oliver_Heaviside). Gibbs sought to convince other physicists of the convenience of the vectorial approach over the [quaternionic](http://en.wikipedia.org/wiki/Quaternion" \o "Quaternion) calculus of [William Rowan Hamilton](http://en.wikipedia.org/wiki/William_Rowan_Hamilton), which was then widely used by British scientists. This led him, in the early 1890s, to a controversy with [Peter Guthrie Tait](http://en.wikipedia.org/wiki/Peter_Guthrie_Tait) and others in the pages of [*Nature*](http://en.wikipedia.org/wiki/Nature_(journal)).[[4]](http://en.wikipedia.org/wiki/Josiah_Willard_Gibbs#cite_note-Bumstead-4)

Gibbs's lecture notes on vector calculus were privately printed in 1881 and 1884 for the use of his students, and were later adapted by [Edwin Bidwell Wilson](http://en.wikipedia.org/wiki/Edwin_Bidwell_Wilson) into a textbook, [*Vector Analysis*](http://en.wikipedia.org/wiki/Vector_Analysis), published in 1901.[[4]](http://en.wikipedia.org/wiki/Josiah_Willard_Gibbs#cite_note-Bumstead-4) That book helped to popularize the "[del](http://en.wikipedia.org/wiki/Del)" notation that is widely used today in [electrodynamics](http://en.wikipedia.org/wiki/Classical_electromagnetism) and [fluid mechanics](http://en.wikipedia.org/wiki/Fluid_mechanics).

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Goertzel, B. (2007). Mirror Neurons, Mirrorhouses, and the Algebraic Structure of the Self. Institute for Ethics and Emerging Technologies.   
<http://ieet.org/index.php/IEET/print/2595> , also <http://www.goertzel.org/dynapsyc/2007/mirrorself.pdf>

**Abstract**. Recent psychological research suggests that the individual human mind may be effectively modeled as involving a group of interacting social actors: both various subselves representing coherent aspects of personality; and virtual actors embodying “internalizations of others.” Recent neuroscience research suggests the further hypothesis that these internal actors may in many cases be neurologically associated with collections of mirror neurons. Taking up this theme, we study the mathematical and conceptual structure of sets of inter-observing actors, noting that this structure is mathematically isomorphic to the structure of physical entities called “mirrorhouses.”

Mirrorhouses are naturally modeled in terms of abstract algebras such as quaternions and octonions (which also play a central role in physics), **which leads to the conclusion that the presence within a single human mind of multiple inter-observing actors naturally gives rise to a mirrorhouse-type cognitive structure and hence to a quaternionic and octonionic algebraic structure as a significant aspect of human intelligence.** Similar conclusions would apply to nonhuman intelligences such as AI’s, we suggest, so long as these intelligences included empathic social modeling (and/or other cognitive dynamics leading to the creation of simultaneously active subselves or other internal autonomous actors) as a significant component.

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Hanson, A. J. (2006). *Visualizing Quaternions*. Morgan Kaufmann

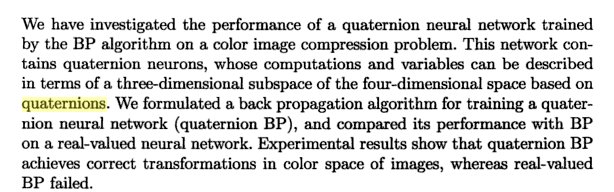
<http://books.google.com/books?id=CoUB09xzme4C&pg=PA69&lpg=PA69&dq=quaternion+lan#v=onepage&q&f=false>

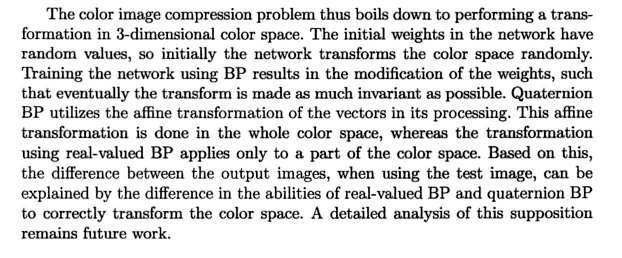
Because quaternions relate three-dimensional coordinate frames to points on a unit hypersphere, it turns out that quaternions provide a meaningful and reliable global framework that we can use to measure the distance or similarity between two different three-dimensional coordinate frames.

Finally, again because they are points on a hypersphere, quaternions can be used to define optimal methods for smooth interpolation among sampled sets of three-dimensional coordinate frames. [*From Preface*]

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Isokawa, T., Kusakabe, T., Nobuyuki, M., Peper, F. (2003). Quaternion neural network and its application. In *Knowledge based intelligent learning and engineering systems, Proceedings, Part II, KES 2003, Oxford, UK,*  *p. 318-324,*Palade, V., Howlett, R. J., Jain, C. (Eds.). *Springer*<http://books.google.com/books?id=BMxSKfVkKskC&pg=PA318&lpg=PA318&dq=big+ball+3D+quaternions+knowledge&source=bl&ots=y-wSW6dvWe&sig=FNOcDYtoSiM9_dDHCgVULOF3GjA&hl=en&sa=X&ei=ExRLVLrDF7SKsQSP1oDQDQ&ved=0CEkQ6AEwBg#v=onepage&q=big%20ball%203D%20quaternions%20knowledge&f=false>





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Jerath, R. and Crawford, M. W. (2014). Neural correlates of visuospatial consciousness in 3D default space: Insights from contralateral neglect syndrome. [Consciousness and Cognition](http://www.sciencedirect.com/science/journal/10538100), 28, 81–93. <http://www.sciencedirect.com/science/article/pii/S1053810014000944>

Summary:

* We propose that the thalamus is a central hub for consciousness.
* We use insights from contralateral neglect to explore this model of consciousness.
* The thalamus may reimage visual and non-visual information in a 3D default space.
* 3D default space consists of visual and other sensory information and body schema.

Abstract:

One of the most compelling questions still unanswered in neuroscience is how consciousness arises. In this article, we examine visual processing, the parietal lobe, and contralateral neglect syndrome as a window into consciousness and how the brain functions as the mind and we introduce a mechanism for the processing of visual information and its role in consciousness. We propose that consciousness arises from integration of information from throughout the body and brain by the thalamus and that the thalamus reimages visual and other sensory information from throughout the cortex in a default three-dimensional space in the mind. We further suggest that the thalamus generates a dynamic default three-dimensional space by integrating processed information from corticothalamic feedback loops, creating an infrastructure that may form the basis of our consciousness. Further experimental evidence is needed to examine and support this hypothesis, the role of the thalamus, and to further elucidate the mechanism of consciousness.

Korsakova-Kreyn, M. (2005). Melodic Rotation, *Universe of Music website.* (Music:Research tab)

<http://universe-of-music.com/melodic-rotation.html>

Musicians habitually think about music in quasi-spatial terms.   
In general, we seem to use spatial abilities in music perception by cognizing melodic objects within tonal reference system (scale). I speculated that the human brain transcends the modality of tonal-temporal patterns in perception of musical structures, and that perhaps same neural substrate is involved in visuo-spatial processing and music cognition. The hypothesis predicted that melodic transformation and mental rotation share same neural substrate in the parietal lobe and that on a certain level of computation our mind transcends the modality of information.   
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Krumhansl, C. L. and Lehrdal, F. (2010) Musical tension. In F. Bacci and D, Melcher (Eds.). Arts and the Senses. Oxford University Press <http://music.psych.cornell.edu/articles/emotion/Bacci%20and%20Melcher-Ch-16.pdf>

From a psychological point of view, the arts can extend our understanding of the

physical, sensory, and perceptual processes that enable us to apprehend the world

around us. Art works are special cases that, at a somewhat higher level, can yield

insights into processes such as memory and attention, and show how knowledge

shapes our interactions with, and interpretations of, artistic objects. At a still higher

level, they raise questions about emotional, social, and cultural factors at work in the

aesthetic realm. Finally, perspectives from neuroscience, genetics, and evolutionary

theory illuminate questions about the underpinnings of artistic creativity.

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Kuipers, J. B. (2002). Quaternions and Rotation Sequences: A Primer with Applications to Orbits, Aerospace and Virtual Reality. Princeton University Press  
<http://www.amazon.com/Quaternions-Rotation-Sequences-Applications-Aerospace/dp/0691102988/ref=sr_1_2?ie=UTF8&qid=1411519227&sr=8-2&keywords=Jack+Kuipers>

## Quaternions are one of the simplest and most powerful tools ever offered to the physicist or engineer. Unfortunately, they are relatively little known because a century-old prejudice (the result of a family feud involving vector theory) has been responsible for keeping them out of university courses. The fact that quaternions have never really found their true role has become a self-fulfilling prophecy, despite their reappearance in various disguised forms such as Pauli matrices, 4-vectors, and, in a complex double form, in the Dirac gamma algebra… [*Peter Rowlands, University of Liverpool, endorsement of book by Jack B. Kuipers*]

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Leclercq, G., Lefevre, P., Blohm, G. (2013). 3D kinematics using dual quaternions: theory and applications in neuroscience. *Frontiers in Behavioral Neuroscience, 7, 7*  
<http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B9>

Our environment is 3-dimensional (3D) and our body can be represented as a rigid multibody system evolving in that 3D environment, with different body parts moving relative to each other. Accurately describing the 3D kinematics of such systems is important for diverse applications in neuroscience which involve 3D kinematics: (1) forward kinematics of different 3D systems: the position and/or orientation of an end-effector (a tool held by the hand for example), (2) 3D Reference frame transformation or reference frame encoding [for example: 3D vestibulor-ocular reflex (VOR), 3D visuomotor transformation, 3D eye-head gaze shifts, 3D spatial updating, …], and (3) inverse kinematics: the set of joint angles corresponding to a given end-effector position and velocity. In practice, these models allow us to analyze 3D data acquired through behavioral experiments as well as to make predictions.

To model the kinematics of a rigid multibody system, we use the framework of the geometric algebra introduced by David Hestenes (see the textbook Hestenes, [1986](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B27)), which easily applies to and is very convenient to model mechanical systems, in particular the kinematics. Hestenes applied this approach to some eye (Hestenes, [1994a](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B28)) and arm reaching (Hestenes, [1994b](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B29)) movements. Actually, he chose the geometric algebra of quaternions which have been sometimes used to model 3D eye, head and arm movements (see for example: (Tweed and Vilis, [1987](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B47); Straumann et al., [1991](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B44); Hore et al., [1992](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B30); Haslwanter, [1995](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B26); Crawford and Guitton, [1997](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B17)). Quaternions allow us to easily deal with rotations but not with translations. Yet we have to deal with translations since the origins of reference frames attached to each body generally do not coincide. For example, the eye rotation center is offset from the head rotation center. **In order to deal with translations, in addition to the rotation, we will use the geometric algebra of dual quaternions (see Bayro-Corrochano,**[**2003**](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B9)**).** Dual quaternions also allow to elegantly represent a screw motion (Funda and Paul, [1990b](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B22); Aspragathos and Dimitros, [1998](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B4)), defined as a combined rotation and translation along the rotation axis. They have been used in Blohm and Crawford ([2007](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B12)) to model combined 3D eye and head rotations in the context of 3D visuomotor transformation for reaching movements. While dual quaternions have not been widely used in the neuroscience community, they have been applied in other fields. Indeed, they have been employed with success in computer vision (Walker et al., [1991](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B48); Phong et al., [1993](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B38); Goddard and Abidi, [1998](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B24); Torsello et al., [2011](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B46)) or in robotics (Daniilidis, [1999](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B19); Bayro-Corrochano et al., [2000](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3576712/#B10)).

The goal of this paper is to provide a tutorial of the dual quaternion geometric algebra to the neuroscientists and then to show its interests and advantages to several applications in sensorimotor control.

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Lehar, S. (2014) The Language of the Mind. Posted, September 12, 2014, Chapter 4 of *The Perceptual Origins of Mathematics. Wordpress book*  
<http://slehar.wordpress.com/2014/09/12/the-language-of-the-mind/>

## The most basic feature of mental imagery is the mental image space, the space of our imagination, which is extended in three dimensions, within which the images of our imagination appear. Attached to this mental image space is a conceptual representation which can be described as an abstract or symbolic code, an array of concept nodes, that represent shapes such as spheres and cubes and pyramids, invariant to their location, orientation, or scale. **The computational function of mental imagery can be described as the top-down projection of an abstract spatial concept into a three-dimensional mental image through an invariance relation,** assigning to the imagined object a specific location, orientation, and scale, by a deliberate act of will.

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Lehar, S. (2014) *Steven Lehar*. Website home page  
<http://cns-alumni.bu.edu/~slehar/Lehar.html>

My latest discovery is Clifford Algebra, a most extraordinary reformulation of all of mathematics that reveals algebra to be actually a branch of geometry. It converts all algebraic relations into spatial relations between spatial structures. It is simultaneously a geometrification of algebra and an algebrification of geometry.

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[David Hestenes](https://physics.asu.edu/people/emeritus-faculty/david-hestenes) (pron. "Hes-ten-es"), who single-handedly rediscovered Clifford Algebra and brought it back to life from complete obscurity, then proceeded to improve on it with two most extraordinary extensions, the projective geometry and the conformal geometry, concepts that are not exclusive to Geometric Algebra, but are very simply expressed in it. **These remarkable extensions reveal the intimate connection between mathematics and perception, revealing mathematics to be an artifact of how our brain makes sense of spatial reality.**

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Behan, K. (2013). Why We Like Sad Music: Part Two - Music, Natural Dog Training, Panksepp and the Constructal Law. Blog Sep 27, 2013  
<http://naturaldogtraining.com/blog/why-we-like-sad-music-part-two/>   
Panksepp, Jaak, quoted by Kevin Behan:

KB: But since animals don’t like being separated, why then do we LIKE sad music?

Panksepp:  “Of course, happiness and sadness work together, and the most moving music allows the two processes to be blended in such a way as to magnify our sense of ourselves as deeply feeling creatures who are conscious inheritors of the tragic view–the ability to see hope and grace in the midst of despair. It may be that the physiological possibility for the experience of chills is established when music joins our deepest opposing emotional potentials within the cradle of consciousness.”

“Separations calls inform parents of the whereabouts of offspring that have become lost, and such calls arouse powerful care-giving motivations in parents, especially the mothers. Internal feeling of coldness and chills when parents hear separation calls may provide increased motivation for social reunion. (body warmth equalling closeness). But why would social loss generate an autonomic response characterized subjectively as chills? Our theoretical view of the underlying evolutionary matters is that the mammalian brain mechanisms for social bonding and feelings of separation distress arose from more ancient, preexisting mechanisms such as those that mediate place attachment, pain perception, and thermoregulatory influences. It is the latter antecedent that may explain the ‘chill’ phenomenon. Social contact giving one the subjective experience of being warmer, whereas social isolation makes one feel colder. . . . (providing) an affective motivational substrate for promoting the types of close interactional patterns that we call social bonds in the absence of direct thermoregulatory stressors.”

[Kevin Behan:] So if I understand this properly, chills make us feel like drawing together in order to get warm and we therefore like chills because sad songs make us feel grateful for the connections that we have, or the kind of closeness that might be possible were we to find those who could satisfy our anxieties of separation.

While I’m very attracted to these kinds of interpretations that root higher cognitive processes to basic bodily processes, I still feel that a deeper explanation can be found in the locomotive circuits because before there were parent/offspring relationships, there were predator and prey relationships, and there was **the absolutely fundamental mandate of moving the body from point A to point B so as to intercept an object of attraction. This would also be more compatible with the research of Dr. Wolpert who states that everything about the brain is dependent on locomotion.** Therefore I believe the emotional affects are not predicated on survival mechanisms or neurological circuitry hardwired in the Central Nervous System, but rather how the body and brain interplay on a much deeper level, specifically how to move the body from point A to point B. Emotional experiences arise in the mind when the  brain generates neurological energies by tracking a subliminal beam of attention on the body’s physical center of gravity, that are likewise coupled to the movement of  external objects of attraction.

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Petoukhov, S.V. (2012). The genetic code, 8-dimensional hypercomplex numbers and dyadic shifts  
<http://arxiv.org/ftp/arxiv/papers/1102/1102.3596.pdf>

[Sergey V. Petoukhov](http://arxiv.org/find/q-bio/1/au:+Petoukhov_S/0/1/0/all/0/1)

*(Submitted on 17 Feb 2011 (*[*v1*](http://arxiv.org/abs/1102.3596v1)*), last revised 30 Jan 2012 (this version, v7))*

**These results testify that living matter possesses a profound algebraic essence. They show new promising ways to develop algebraic biology.**

(repeated below in context) ….

Matrix forms of the representation of the multi-level system of molecular-genetic alphabets have revealed algebraic properties of this system. **Families of genetic (4\*4)- and (8\*8)-matrices** show an unexpected connections of the genetic system with functions by Rademacher and Walsh and with Hadamard matrices. **Dyadic-shift decompositions of such genetic matrices lead to relevant algebras of hypercomplex numbers.** It is shown that genetic Hadamard matrices are **identical to matrix representations of Hamilton quaternions** and its complexification in the case of unit coordinates. **The diversity of known dialects of the genetic code is analyzed from the viewpoint of the genetic algebras.** An algebraic analogy with Punnett squares for inherited traits is shown. Our results are discussed taking into account the important role of dyadic shifts, Hadamard matrices, fourth roots of unity, Hamilton quaternions and other hypercomplex numbers in mathematics, informatics, physics, etc. **These results testify that living matter possesses a profound algebraic essence. They show new promising ways to develop algebraic biology.**

…

In the beginning of the XIX century the following opinion existed: the world possesses

the single real geometry (Euclidean geometry) and the single arithmetic. But this opinion was

rejected after the discovery of non-Euclidean geometries and quaternions by Hamilton. The

science understood that **different natural systems can possess their own individual geometries**

and their own individual algebras (see this theme in the book [Kline, 1980])…

The fact, that the genetic code has led us to DS-algebras of 8-dimensional hypercomplex

numbers (first of all, R-octenons and H-octetons), **testifies in favor of the importance of these**

**algebras for organisms. It seems that many difficulties of modern science to understand genetic**

**and biological systems are determined by approaches to these systems from the viewpoint of**

**non-adequate algebras, which were developed entirely for other systems.** In particular, the

classical vector calculation, which plays the role of the important tool in classical mechanics and

which corresponds to geometrical properties of our physical space, can be inappropriate for

many biological phenomena. One can think that living substance lives in its own biological space

which has specific algebraic and geometric properties. Genetic octetons, which are described in

this article, can correspond to many aspects in such biological space. In comparison with our

physical space and its vector calculation, vector spaces of genetic octetons are related with

another type of vector calculations, etc.

Here it can be remind that E. Schrodinger thought that gaining knowledge about a

“stream of order” in living matter is a critical task. He wrote in his book [Schrodinger, 1992,

Chapter VII]: “What I wish to make clear in this last chapter is, in short, that from all we have

learnt about the structure of living matter, we must be prepared to find it working in a manner

that cannot be reduced to the ordinary laws of physics. And that not on the ground that there is

any “new force” or what not, directing the behavior of the single atoms within a living

organism, but because the construction is different from anything we have yet tested in the

physical laboratory... The unfolding of events in the life cycle of an organism exhibits an

admirable regularity and orderliness, unrivalled by anything we meet with in inanimate matter…

To put it briefly, we witness the event that existing order displays the power of maintaining itself

and of producing orderly events… We must be prepared to find a new type of physical law

prevailing in it /living matter/”.

The hypothesis about a non-Euclidean geometry of living nature exists long ago

[Vernadsky, 1965] but without a concrete definition of the type of such geometry. Our results

draw attention to concrete multidimensional vector spaces which have arised in researches of the

genetic code. All the described facts provoke the high interest to the question: what is life from

the viewpoint of algebra and geometry? This question exists now in parallel with the old

question from the famous book by E.Schrodinger: what is life from the viewpoint of physics?

One can add that attempts are known in modern theoretical physics to reveal information bases

of physics; in these attempts information principles are considered as the most fundamental.

Mathematics has deals not only with algebras of numbers but with algebras of operators also

(see historical remarks in the book [Kline,1980, Chapter VIII]). G.Boole has published in 1854

year his brilliant work about investigations of laws of thinking. He has proposed the Boole’s

algebra of logics (or logical operators). Boole tried to construct such operator algebra which

would reflect basic properties of human thinking. The Boole’s algebra plays a great role in the

modern science because of its connections with many scientific branches: mathematical logic,

the problem of artificial intelligence, computer technologies, bases of theory of probability, etc.

**In our opinion, the genetic DS-algebra of octetons can be considered not only as the algebras**

**of the numeric systems but as the algebra of proper logical operators of genetic systems also.**

**This direction of thoughts can lead us to deeper understanding of logic of biological systems**

**including an advanced variant of the idea by Boole (and by some other scientists) about**

**development of algebraic theory of laws of thinking. One can add that biological organisms**

**possess known possibilities to utilize the same structures for multi-purpose destinations. The**

**genetic DS-algebras can be utilized by biological organisms in different purposes also.**  
  
…

In a general case, numeric coordinates of the Hadamard spectrum ū (see expression (11))

are quite different from numeric coordinates of the initial vector ā = [a0; a1; a2; a3; a4; a5; a6; a7].

In contrast to this case, coordinates of hk-spectrums ūk (15) are always taken from the same set of

coordinates a0; a1; a2; a3; a4; a5; a6; a7 of the initial vector ā (only their signs “+” and “–“ can be

changed). In other words, all the hk-spectral operations are special cases of permutation operators

which not only make a permutation of coordinates of 8-dimensional vectors but also invert sign

of some coordinates. Such kind of spectral analysis can be conditionally called the “genetic

permutation spectral analysis”. In a general case of such analysis, **8-dimensional vectors ā can**

**contain not only real numbers but also many other objects** if their sequences are presented in a

form of 8-dimensional vectors: genetic triplets and their associations, letters, phrases, complex

and hypercomplex numbers, matrices, photos, **musical notes and fragments of musical works,**

etc. From the linguistic viewpoint, hk-spectrums are anagrams to within plus or minus sign of

entries.  
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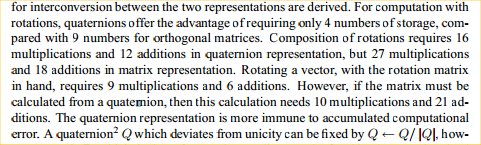
Piaget, Jean

Evans: Why do you think that mathematics is so important in the study of the development of knowledge?  
Piaget: Because, along with its formal logic, mathematics is the only entirely deductive discipline. Everything in it stems from the subject's activity. It is man-made. What is interesting about physics is the relationship between the subject's activity and reality. What is interesting about mathematics is that it is the totality of what is possible. And of course the totality of what is possible is the subject's own creation. That is, unless one is a Platonist.

*From a 1973 interview with Richard Evans (Jean Piaget: The Man and His Ideas)*

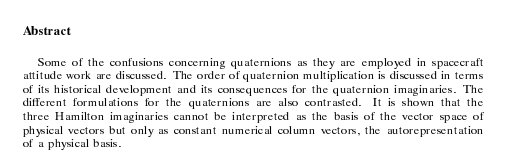
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Salamin, E. (1979) Application of Quaternions to Computation with Rotations. Working paper, Stanford AI Lab  
<http://www.theworld.com/~sweetser/quaternions/ps/stanfordaiwp79-salamin.pdf>



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Shuster, M. D. (2008). The nature of the quaternion. Journal of the Aeronautical Sciences, 53, 3, 359-373. For draft revision 1g, see:  
<http://www.malcolmdshuster.com/pubp_021_072x_j_jas0000_quat_mds.pdf>



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Sangwine, S. J. and Ell, T. A. (2000). Colour image filters based on hypercomplex convolution. *IEEE Proceedings – Vision, Image, and Signal Processing, 147, 2*, 89-93  
<https://www.researchgate.net/publication/3359119_Colour_image_filters_based_on_hypercomplex_convolution>

**ABSTRACT** There are very few examples of true vector filters known for  
colour images. The authors introduce a new class of filter based on  
convolution with hypercomplex masks, and present three colour edge  
detecting filters inspired by the Prewitt, Sobel and Kirsch filters.  
These filters, when applied to a colour image, produce an almost  
greyscale image with colour edges where the original image had a sharp  
change of colour. They rely on a three-space rotation about the grey  
line of RGB space, implemented with the rotation operator R[]R-1where R is a quaternion and R-1 is its inverse. Pixels  
of similar colour corresponding to opposing positions in the filter  
masks cancel to give a grey or monochromatic result, while pixels of  
different colour are differenced in a vector sense to give coloured edge  
pixels. The linearity of the new filters is discussed and the paper  
concludes with a discussion of the impulse response of a hypercomplex  
filter

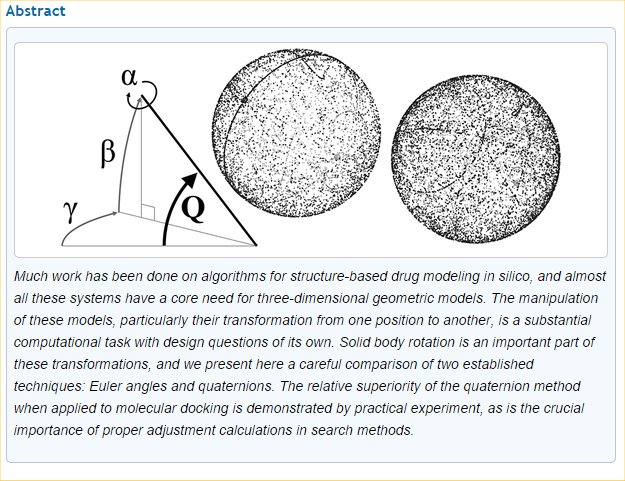
*Background note [from University of Essex]:*

Research on hypercomplex colour image processing started in 1996 and arose from the work of Dr. Amy Thornton who applied complex Fourier transforms to chrominance images ([see here](http://privatewww.essex.ac.uk/~sjs/research/colour_ip.html#amy)). The quaternions are a generalization of the complex numbers with an imaginary part consisting of three components, and the idea was to use these to generalize techniques from signal and image processing, such as frequency domain filtering and correlation, to handle colour images as vector images. More details about quaternions and links to general information elsewhere are given on a [separate page](http://privatewww.essex.ac.uk/~sjs/research/quaternions.html).

The first development, in 1996, was a quaternion Fast Fourier Transform (FFT) based on the work of [Dr Todd A. Ell](http://home.att.net/~t.a.ell/) at the University of Minnesota and published in 1993. Subsequently a collaboration has developed and recently (June 2003) this collaboration has been supported by the award of an EPSRC Visiting Fellowship (Grant no GR/S58621/01) valid for 3 years. In 1998, a new colour edge detector was developed using quaternion convolution. The result of applying this edge detector to the Lena image is shown on a [separate page](http://privatewww.essex.ac.uk/~sjs/research/colour_edge_images.html).

[Above text from: <http://privatewww.essex.ac.uk/~sjs/research/hypercomplex_ip.html> ]

Skone, G., [Cameron](http://pubs.acs.org/action/doSearch?action=search&author=Cameron%2C+S&qsSearchArea=author), S., and [Voiculescu](http://pubs.acs.org/action/doSearch?action=search&author=Voiculescu%2C+I&qsSearchArea=author), I.(2013). Doing a Good Turn: The Use of Quaternions for Rotation in Molecular Docking. J. Chemical Information and Modelling (ACS), 53(12), 3367-3372  
<http://pubs.acs.org/doi/abs/10.1021/ci4005139>



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Sylvester, James.   
<http://www.ualr.edu/lasmoller/matrices.html>

"Mathematics is the music of reason."

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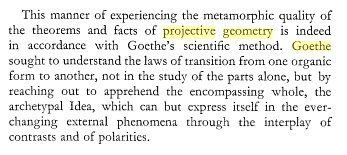
Trehub, A. (2013) Where Am I? Redux. *J. of Consciousness Studies, 20, (1-2)*, 207-225  
<http://www.ingentaconnect.com/content/imp/jcs/2013/00000020/F0020001/art00011>

### Abstract:

Activation of the brain's putative retinoid system has been proposed as the neuronal substrate for our basic sense of being centred within a volumetric surround -- our minimal phenomenal consciousness (Trehub, 2007). Here, the assumed properties of the self-locus within the retinoid model are shown to explain recent experimental findings relating to the out-of-body experience. In addition, selective excursion of the heuristic self-locus is able to explain many important functions of consciousness, including the effective internal representation of a 3D space on the basis of 2D perspective depictions. Our sense of self-agency is shown to be a natural product of the role of the heuristic self-locus in the retinoid mechanism.

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Whicher, Olive (1971). *Projective geometry; creative polarities in space and time*. Rudolf Steiner Press  
<http://books.google.com/books/about/Projective_geometry_creative_polarities.html?id=bzoZAQAAIAAJ>



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### White, S. (2014). Geometry of Quaternions. Web pages on quaternions at www.Zipcon.net <http://www.zipcon.net/~swhite/docs/math/quaternions/geometry.html> **“Geometry in different dimensions**

Four-space has its own geometry, but in the context of quaternions, most of the geometrical literature is concerned with how quaternions affect three-space as operations.

### **Change of direction and rotation**

The quaternions are often described as a “change of direction and rotation”, by which is meant, a general transformation of one 3-vector into another.

Is the dimensionality right? Consider what information is needed to transform one 3-vector into another.

* First, the unit normal to the plane containing the origin and both the vectors. That requires two degrees of freedom (a point on the unit sphere in 3-space).
* Then the angle between the two vectors
* Finally, the ratio of the lengths of the vectors.

Four degrees of freedom, altogether.

The quaternions are a four-dimensional space, so they should be able to model such a transformation.”

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Whyte, L.L. (1954). A dimensionless physics? *The British Journal for the Philosophy of Science, 5, 17*, 1-17  
<http://www.jstor.org/stable/685948>

L.L. Whyte on Quaternions (and their value in moving toward a dimensionless physics):

*(pages 11-12) – accessed via JSTOR.*

Eddington's desire to move towards a dimensionless theory was frustrated by his failure to observe that a genuinely dimensionless theory, incorporating the measures and the interactions within the system, requires that linear metric must be eliminated from the postulates. Thus he was left in the confused position of seeking to maintain two propositions simultaneously: quantum theory is a pure number theory and a linear metric is implicit in its physical interpretation, for example of the quantum conditions. The second is correct, but the first is a misleading half-truth.

**(f) Many workers have considered the relation of quaternions to special relativity and to relativistic quantum theory. If a quaternion is defined, following Hamilton's first method, as a dimensionless quotient of two vectors (lines possessing length, orientation, and sense), the introduction of quaternions may be regarded as a step towards a dimensionless theory.** We can interpret Tait's cry,' Repent Cartesian sins and embrace the true faith of quaternions ! ' as meaning 'Drop lengths and substitute angles ! ' Kilmister ' has shown that Eddington's formulation of Dirac's equations can be simplified by using quaternions, and interpreted as representing the non-metrical properties of an affine space of distant parallelism. Thus Dirac's equations in Kilmister's derivation are independent of metric.

Notes:

1. A. S. Eddington, The Philosophy of Physical Science, Cambridge, 1939, p. 75.   
   See also Fundamental Theory, Cambridge, 1946, p. 7
2. A. S. Eddington, Fundamental Theory, p. 14
3. For references see C. W. Kilmister, Proc. Roy. Soc., A, 1949, 199, 517

*[This content downloaded from 98.14.30.168 on Fri, 11 Apr 2014 02:27:28 AM. All use subject to JSTOR Terms and Conditions – “A DIMENSIONLESS PHYSICS?” ]*

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Wikipedia (2014). Four-dimensionalism. *See “Comparisons to three-dimensionalism*”  
<http://en.wikipedia.org/wiki/Four-dimensionalism>

Comparisons to three-dimensionalism:

Unlike the four dimensionalist, the three dimensionalist considers time to be a unique [dimension](http://en.wikipedia.org/wiki/Dimension) that is not analogous to the three spatial dimensions: [length](http://en.wikipedia.org/wiki/Length), [width](http://en.wikipedia.org/wiki/Width) and [height](http://en.wikipedia.org/wiki/Height). Whereas the four dimensionalist proposes that objects are extended across time, the three dimensionalist adheres to the belief that all objects are wholly present at any moment at which they exist. While the three dimensionalist agrees that the parts of an object can be differentiated based on their spatial dimensions, they do not believe an object can be differentiated into temporal parts across time. For example, in the three dimensionalist account, "Descartes in 1635" is the same object as "Descartes in 1620", and both are identical to Descartes, himself. However, the four dimensionalist considers these to be distinct temporal parts, neither of which are identical with the whole Descartes, the [spacetime](http://en.wikipedia.org/wiki/Spacetime" \o "Spacetime) "worm" concatenating these temporal parts.[[6]](http://en.wikipedia.org/wiki/Four-dimensionalism#cite_note-3D.2F4D_debate-6)

Three Dimensions in Indigenous Philosophy

1. Personal 2. Communal 3. Universal

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Wolpert, D. (2011). The Real Reason for Brains. TEDGlobal2011. Video Presentation, 20 min.  
<http://www.ted.com/talks/daniel_wolpert_the_real_reason_for_brains?language=en>

TEDGlobal 2011 (20-min. video):

Neuroscientist Daniel Wolpert starts from a surprising premise: the brain evolved, not to think or feel, but to control movement. In this entertaining, data-rich talk he gives us a glimpse into how the brain creates the grace and agility of human motion.

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Wolpert, D. M., Goodbody, S. J., Husain, M. (1998). Maintaining internal representations: the role of the human superior parietal lobe. *Nature, 1,6,*   
<http://www.nature.com/neuro/journal/v1/n6/full/nn1098_529.html>

In sensorimotor integration, sensory input and motor output signals are combined to provide an internal estimate of the state of both the world and one's own body. Although a single perceptual and motor snapshot can provide information about the current state, computational models show that the state can be optimally estimated by a recursive process in which an internal estimate is maintained and updated by the current sensory and motor signals. These models predict that an internal state estimate is maintained or stored in the brain. Here we report a patient with a lesion of the superior parietal lobe who shows both sensory and motor deficits consistent with an inability to maintain such an internal representation between updates. Our findings suggest that the superior parietal lobe is critical for sensorimotor integration, by maintaining an internal representation of the body's state.

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A.P. Yefremov (2005). Quaternions and biquaternions: algebra, geometry, and physical theories. Arxiv.org  
<http://arxiv.org/pdf/math-ph/0501055.pdf>

* At the beginning of [the] 20[th] century [the] last bastion of Q-numbers amateurs, ”Association for the Promotion of the Study of Quaternions”, was ruined. The only reminiscence of once famous hypercomplex numbers was the set of Pauli matrices.
* Later on quaternions appeared incidentally as a mathematical means for alternative description of already known physical models [6, 7] or due to surprising simplicity and beauty they were used to solve rigid body cinematic problems [8].
* **An interest to quaternionic numbers essentially increased in last two decades when a new generation of theoreticians started feeling in quaternions deep potential yet undiscovered (e. g. [9] – [11]).**

…

One can say that space-time model and kinematics of the Quaternionic Relativity are nowadays studied in enough details and can be used as an effective mathematical tool for calculation of many relativistic effects. But respective relativistic dynamic has not been yet formulated, there are no quaternionic field theory; Q-gravitation, electromagnetism, weak and strong interactions are still remote projects.   
  
However, there is a hope that it is only beginning of a long way, and the theory will ”mature”. This hope is supported by observation of number of remarkable ”Quaternionic Coincidences” forming a discrete mosaic of physical and mathematical facts; **probably one day it will turn into a logically consistent picture providing new instruments and extending our insight of physical laws.**